Lecture 29

11.9 - Representing Functions as Power Series

Starting Point

From our knowledge of geometric series, we have a power series representation of $g(x) = \frac{1}{1-x}$ as:

Using this as our model, we can represent more functions as power series:

Ex: Represent the following functions as power series and find the interval of convergence:

ⓐ $f(x) = \frac{2}{4-x}$

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(b) $g(x) = \dfrac{x^2}{1+x}$

(c) $h(x) = \dfrac{-3}{x^2+2x}$

Differentiation & Integration of Power Series

Just as with normal polynomials, we can differentiate and integrate a power series term-by-term.

Theorem: If the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ has radius of convergence R then the function f defined by

$$f(x) = c_0 + c_1 x + c_2 x^2 + \cdots = \sum_{n=0}^{\infty} c_n x^n$$

is differentiable on the interval $(a-R, a+R)$ with

$$f'(x) =$$

and is also integrable with

$$\int f(x)\,dx =$$

The radius of convergence for both of these series is R. Regardless of the convergence at the endpoints for $f(x)$, after integrating or differentiating, convergence at endpoints should be checked again.

Ex: Find a power series representation of

$$f(x) = \frac{1}{(x+1)^2}$$

Ex: Find a power series representation of $g(x) = \arctan(x)$.

Ex: Find the sum of the series:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(\sqrt{3})^{2n+1}(2n+1)}$$